Lecture 17

Wednesday, October 19, 2016 9:04 AM

4.5 Summary of Curve Sketching:

A. Domain
B. Intercepts
C. Symmetry
D. Asymptotes
E. Intervals of Increase/Decrease
F. Local Max/Min
G. Concavity/ Inflection Points
H. **Sketch the Curve**

Ex Sketch the curve $y = \frac{2x^2}{x^2-1}$

A. Domain

$x^2 - 1 \neq 0 \Rightarrow x^2 \neq 1 \Rightarrow x \neq \pm 1$

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

B. Intercepts

y-int is when $x = 0$, $y = \frac{2 \cdot 0^2}{0^2 - 1} = 0$

x-int is when $y = 0$, $0 = \frac{2x^2}{x^2-1}$

$\Rightarrow 2x^2 = 0 \Rightarrow x = 0$

Both x and y int are $(0,0)$

C. Symmetry: $f(x) = \frac{2x^2}{x^2-1}$

$f(-x) = \frac{2(-x)^2}{(-x)^2 - 1} = \frac{2x^2}{x^2-1} = f(x)$

Even function.

D. Asymptotes

H.A Need to find $\lim_{x\to\infty} f(x)$ & $\lim_{x \to -\infty} f(x)$

$$\lim_{x\to\infty} \frac{2x^2}{x^2-1} = \lim_{x\to\infty} \frac{\frac{2x^2}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{2}{1 - \frac{1}{x^2}} = \frac{2}{1} = 2$$

$$\lim_{x\to -\infty} \frac{2x^2}{x^2-1} = 2.$$

V.A $x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1.$

Check that $\lim_{x \to \pm 1^{\pm}} f(x) = \pm \infty$

$$\lim_{x\to 1^+} \frac{2x^2}{x^2-1} = +\infty$$

E. Intervals of Increase / Decrease

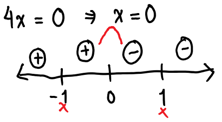
$$y = \frac{2x^2}{x^2-1}, \quad \frac{dy}{dx} = \frac{(x^2-1)4x - 2x^2(2x)}{(x^2-1)^2}$$

$$= \frac{4x^3 - 4x - 4x^3}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{-4x}{(x^2-1)^2}$$

$(x^2-1)^2 = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$

$-4x = 0 \Rightarrow x = 0$



Decreasing on $(0,1) \cup (1,\infty)$

Increasing on $(-\infty,-1) \cup (-1,0)$

F. Local Max / min

By first derivative test local max

@ $x = 0$, $(0,0)$

G. Concavity / Inflection Points

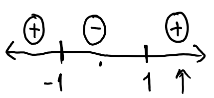
$$\frac{dy}{dx} = \frac{-4x}{(x^2-1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x^2-1)^2 \cdot (-4) - (-4x)\,2(x^2-1) \cdot 2x}{(x^2-1)^4}$$

$$= \frac{\cancel{(x^2-1)}\left[(x^2-1) \cdot (-4) + 4x \cdot 4x\right]}{(x^2-1)^{\cancel{4}\,3}}$$

$$= \frac{-4x^2 + 4 + 16x^2}{(x^2-1)^3} = \frac{12x^2+4}{(x^2-1)^3}$$

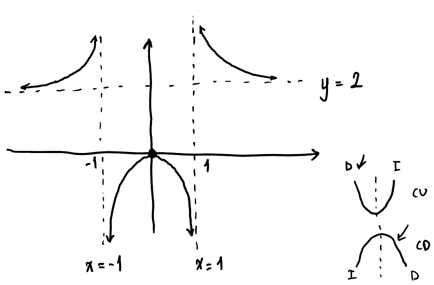
$\frac{d^2y}{dx^2} \neq 0$, $x = \pm 1$ where $\frac{d^2y}{dx^2}$ not def.



C.U $(-\infty,-1) \cup (1,\infty)$

C.D $(-1,1)$

-1, 1 are not I.P since they are not in the domain of the function.



Ex $f(x) = \frac{\cos x}{2 + \sin x}$

A.

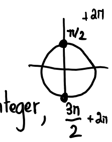
Domain : $2 + \sin x \neq 0 \Rightarrow \sin x \neq -2$

Since $-1 \leq \sin x \leq 1$, $\sin x$ is never equal to 2, hence domain is $(-\infty,\infty)$

B. Intercepts

Y-intercepts $x = 0$, $y = \dfrac{\cos 0}{2+\sin 0} = \dfrac{1}{2}$ $\left(0, \frac{1}{2}\right)$

x-int $y = 0$, $0 = \dfrac{\cos x}{2+\sin x} \Rightarrow \cos x = 0$



$x = \dfrac{\pi}{2} + 2n\pi$, n is an integer,

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$= \dfrac{3\pi}{2} + 2n\pi$,

C. Symmetry

$f(x) = \dfrac{\cos x}{2+\sin x}$, $f(-x) = \dfrac{\cos(-x)}{2+\sin(-x)} = \dfrac{\cos x}{2-\sin x} \neq f(x)$

$\neq -f(x)$

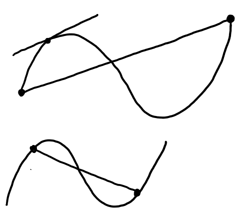
$f(x+2\pi) = \dfrac{\cos(x+2\pi)}{2+\sin(x+2\pi)} = \dfrac{\cos x}{2+\sin x} = f(x)$

We only need to worry about $[0, 2\pi]$

f is cont on $[a,b]$ $\ln x$ on [2

diff on (a,b)

c betn a & b s.t $f'(c) = \dfrac{f(b)-f(a)}{b-a}$



$\lim_{x \to 1} \dfrac{2x^2}{x^2-1} = \dfrac{2}{0}$

$\dfrac{2}{\to 0}$

$\lim_{x \to 1^+} \dfrac{2x^2}{x^2-1} \overset{\frac{+}{+}}{=} +\infty$

$\lim_{x \to 1^-} \dfrac{2x^2}{x^2-1} = -\infty$